Nonlinear Predictive Controller for the Simplified ABB Test Power System Stabilization Problem

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Abstract

A nonlinear predictive controller is proposed to solve the simplified ABB test problem. Interesting feature of the proposed control law is its ability to be easily generalizable to the original problem. Many validating scenarios are proposed to illustrate the effectiveness of the proposed approach.

1 Introduction

The ABB benchmark is a power system stabilization problem that has been defined in the context of the CC-European project in order to illustrate control strategies on hybrid systems.

The preliminary studies on the above benchmark have showed high sensitivity of open-loop behaviours to control parameterization (at least for the control strategies that have been tested). This suggested the use of a more simplified model, at least in a first step, in order to gain a deeper insight into both the simplified and the original problem.

The work proposed in this paper treats the simplified test model problem that has been proposed in [2] as an issue to the Ascona meeting in October 2002. In the present work, it is shown that by applying a simple nonlinear predictive controller, it is possible to find open-loop strategies that steers the simplified system (that has been destabilized after a change in the transmission network’s reactance) while being compatible with the tap changer discrete mechanism as well as the available computation time. The resulting steering open-loop control profiles are then used in a predictive control scheme yielding a stabilizing feedback controller.

The present report is organized as follows. First, the equations of the simplified model are briefly recalled in section 2. Then the proposed algorithm is detailed in section 3. Some related implementation issues that may be needed for an efficient implementation of the proposed strategies are then discussed in section 4. Finally, section 5 shows numerical simulations illustrating the effectiveness of the proposed algorithm in stabilizing the simplified model while handling the relatively non standard control specifications.

2 The simplified model [2]

The simplified system considered in this report is depicted on figure 1. According to [2, 1], focusing on tap changer dynamics (slow when compared to the generator dynamics), the
The generator bus can be modelled as an infinite bus. The load is modelled as an exponential recovery load, namely

\[ \dot{x}_p = -\frac{x_p}{T_p} + P_0(1 - v^2) \]

\[ \dot{x}_q = -\frac{x_q}{T_q} + Q_0(1 - v^2) \]

\[ P_d = (1 - k)(x_p/T_p + P_0v^2) \]

\[ Q_d = (1 - k)(x_q/T_q + Q_0v^2) \]

where \( P_d \) is the actual active load power, \( Q_d \) the actual reactive load power while \( T_d \) and \( T_q \) are the corresponding recovery time constants. The transmission network is simply modelled as a pure reactance in series with an ideal transformer. Note that the parameter \( (1 - k) \) has been introduced to enable load shedding to be modelled. Note then that \( k \) must satisfy the following saturation-type constraint

\[ 0 \leq k \leq k_{\text{max}} < 1 \]

more precisely, \( k \) is assumed to belong to the following discrete set

\[ k \in \mathcal{K} := \{0, 0.05, 0.1, \ldots k_{\text{max}}\} \]

**Remark 1** In the original simplified problem \( k_{\text{max}} = 0.15 \) is considered. In the simulations of the present paper, we take \( k_{\text{max}} = 1 \) since this enables to consider a more general definition of the constraint. If in a simulation, \( k \) takes values that are greater than 0.15, one just has to consider that under the original constraint definition, the voltage would collapse.

Now, the active and the reactive powers supplied to the load are clearly given by [2]

\[ P_d = -\frac{v_0 v}{nX} \sin \delta \]

\[ Q_d = \frac{v_0 v}{nX} \cos \delta - \frac{v^2}{n^2 X} + \mu B_0 v^2 \]

where the term \( \mu B_0 v^2 \) represents the reactive power in the capacitor bank (see figure 1). Note that \( B_0 \) is a constant while \( \mu \) is a control action representing relative change in the capacitor bank resulting value. The control input \( \mu \) is constrained to belong to the following discrete set\(^1\)

\[ B_0 = 0.2 \quad ; \quad \mu \in \mathcal{M} := \{0.25, 0.75, 1, 1.25, 1.5\} \]

\(^1\)This choice is mandatory and the proposed algorithm holds for any choice of such a discrete set. The set defined in (10) while realistic is only an illustrative choice.
this clearly leads to the following discrete set of admissible capacitors

$$\mu B_0 \in \{0.05, 0.15, 0.2, 0.25, 0.3\}$$  \hspace{1cm} (11)$$

The resulting DAE’s model is then obtained by writing equalities between (3) and (7) in one hand and between (4) and (8) in the other hand. More precisely, using the following notations

$$x := \begin{pmatrix} x_p \\ x_q \end{pmatrix} ; \quad y := v ; \quad u := \begin{pmatrix} n \\ k \end{pmatrix}$$  \hspace{1cm} (12)$$

the system equations can be written as follows

$$\dot{x} = f(x, y) := \left( -\frac{x_p}{T_p} + P_0(1 - y^2) \right)$$
$$0 = g(x, y, u) := \left( -\frac{x_q}{T_q} + Q_0(1 - y^2) \right)$$  \hspace{1cm} (13)

$$0 = \mu B_0 y^2 \sin \delta + \left( 1 - k \right) \left( \frac{x_p}{T_p} + P_0 y^2 \right)$$
$$0 = \mu B_0 y^2 \cos \delta + \frac{y^2}{n^2} - \mu B_0 y^2 + \left( 1 - k \right) \left( \frac{x_q}{T_q} + Q_0 y^2 \right)$$  \hspace{1cm} (14)

It is worth noting that because of the tap changer dynamics, the transformer ratio \(n\) cannot be rigorously considered as a control variable. The true control variable must be \(v_r\) that is used in the tap changer definition (see figure 2). However, it is clear that if the following three conditions are respected

- The open-loop control input \(n(.)\) is piece-wise constant with a sampling period \(T_d\) (where \(T_d\) is the time delay of the OLTC model)
- The piece-wise constant open-loop control input \(n(.)\) is such that for all \(j \in \mathbb{N}\), one has
  $$|n(j + 1) - n(j)| = d_n \quad (= 0.02)$$
  where \(d_n\) is the step size modulus applied to \(n\) by the OLTC model when entering the "action" mode.
- The piece-wise constant open-loop control input \(n(.)\) is such that for all \(j \in \mathbb{N}\), one has
  $$0.8 = n_{\text{min}} \leq n(j) \leq n_{\text{max}} \quad (= 1.2)$$

then it is possible to use \(n\) as a constrained control variable and to obtain \(v_r\) by approximately inversing the dynamic of \(n\). This is done by taking

$$\forall t \in [jT_d, (j + 1)T_d[, \hspace{0.5cm} v_r(t) = v(t) + u_f \times \text{Sign}(n(j + 1) - n(j))$$  \hspace{1cm} (15)$$

**Definition 1** A sequence \(\left( n(j) \right)_{j \in \mathbb{N}}\) that meets the above three requirements is said to be tap-changer compatible.

### 3 A nonlinear predictive controller

Nonlinear predictive control [3] is now widely recognized to be a feedback strategy providing a relatively easy handling of both nonlinearity, constraints and optimality. Recall that predictive control scheme amounts to compute at each sampling time an optimal open-loop control sequence (in the sense of some given cost function), to apply the first part of the resulting optimal open-loop control sequence until the next sampling instant. At the next sampling
WaitCountAction
WaitCountAction
WaitCountAction

Figure 2: The non sequential OLTC control system

instant, the whole problem is re-considered on a moving-horizon and the procedure is repeated indefinitely resulting in a state feedback law.

For nonlinear systems and for long prediction horizons, this may lead to open-loop optimal control problems with a high dimension of the decision variable. This joined to the non convex nature of the problem may render the on-line computations necessary to implement the resulting strategy unfeasible.

That is the reason why a key feature in nonlinear predictive control design lies in the choice of control parameterization that leads to reasonably reduced complexity. The aim of the following subsection is to clearly define a reduced dimensional parameterization of open-loop controls that are used afterward in the predictive control implementation. Recall that a key feature in receding-horizon control is that the resulting closed-loop control is much more rich than the underlying open-loop parameterization. The consequence of this is that in many cases, apparently over-simplified open-loop parameterizations results in a sufficiently rich closed-loop control behaviour.

3.1 The open-loop control parameterization

Recall that the control is implemented in a sampling scheme with a sampling period \( \tau = T_d \) and that based on this assumption, the control input in our problem is given at each sampling instant \( j \tau \) by

\[
 u(j) := \begin{pmatrix} n(j) \\ \mu(j) \\ k(j) \end{pmatrix} \in \left\{ n(j - 1) - d_n , n(j - 1) , n(j - 1) + d_n \right\} \times \mathcal{M} \times \mathcal{K}
\]  

(16)

with the constraint \( n_{\text{min}} \leq n(j) \leq n_{\text{max}} \). Recall that the sets \( \mathcal{M} \) and \( \mathcal{K} \) have been defined in (10) and (6) respectively.

**Remark 2** It is needless to say that the sampling period \( \tau \) may be different for the three control component. This enables \( k \) and \( \mu \) to be more reactive than \( n \) restricted by the tap changer dynamics. In this study, we preferred to adopt a unified sampling scheme for simplicity.

Like any predictive control scheme, one must first define a prediction horizon, that is, the future time horizon over which some cost function is to be minimized. Given some sampling
time, this prediction horizon may be defined as an integer number $N_p$ of sampling periods (see Figure 3). To define the control parameterization, one has to clearly define at each sampling instant $j\tau$ the structure of the control $u(\cdot)$ over the time horizon $[j\tau, (j + N_p)\tau]$. Before, let us use the following notations to denote the open-loop control profiles at instant $j\tau$ over the prediction horizon $[j\tau, (j + N_p)\tau]$:

\[ \tilde{u}(j) := (u(j\tau), \ldots, u((j + N_p - 1)\tau)) \quad ; \quad \tilde{n}(j) := (n(j\tau), \ldots, n((j + N_p - 1)\tau)) \]

\[ \tilde{\mu}(j) := (\mu(j\tau), \ldots, \mu((j + N_p - 1)\tau)) \quad ; \quad \tilde{k}(j) := (k(j\tau), \ldots, k((j + N_p - 1)\tau)) \]

The control parameterization is defined by adopting the following choices

- The control profiles $\tilde{k}(j)$ and $\tilde{\mu}(j)$ are constant.
- The control profiles $\tilde{n}(j)$ is monotonic starting from $n(j - 1)$
- The control profiles $\tilde{n}(j)$ is constant on $[(j + N_c)\tau, (j + N_p)\tau]$

The last two points define $2N_c + 1$ possible choices for $\tilde{n}(j)$. $N_c$ increasing profiles, $N_c$ decreasing profiles and 1 constant profiles. As a result, the resulting parameterization leads to a decision variable of dimension

\[ (2N_c + 1) \cdot \text{card}(\mathcal{M}) \cdot \text{card}(\mathcal{K}) \]

Note also that all candidate sequences $\tilde{n}(j)$ defined by the above rules are tap changer-compatible in the sense of definition 1.

### 3.2 Further definitions and notations

In order to properly present the optimal control computation, some further definitions and notations are needed.

Let $\tilde{U}(n(j - 1))$ be the set of admissible profiles at instant $j\tau$. The fact that this set depends on the past value of $n$ results from (16). This will be shortly denoted by $\tilde{U}_j$. Sometimes, the index $j$ is omitted when no ambiguity follows.
We shall define an equivalence relation on $\tilde{U}_j$ by
\[
\left\{ \tilde{u}^{(1)} \sim \tilde{u}^{(2)} \right\} \Leftrightarrow \left\{ (\tilde{\mu}^{(1)}, \tilde{k}^{(1)}) = (\tilde{\mu}^{(2)}, \tilde{k}^{(2)}) \right\}
\] (18)

The need for such equivalence relation comes from the fact that while constraints are imposed on the use of capacitor changes ($\mu$) or load shedding ($k$), no explicit constraint is considered to state that such tap changer compatible sequence $\tilde{n}^{(1)}$ is better or worst that some other one, still compatible sequence $\tilde{n}^{(2)}$.

Denote by
\[
U_{eq}^j := \left\{ \tilde{U}^{(\mu, k)}_j \right\}_{(\mu, k) \in M \times K}
\]
the set of equivalence classes that partitions $\tilde{U}_j$ according to the equivalence relation (18), namely, an equivalent class (an element of $U_{eq}^j$) is defined by the corresponding $(\mu, k)$ pair
\[
\tilde{U}^{(\mu, k)}_j := \left\{ \tilde{u} = (\tilde{n}, \tilde{\mu}, \tilde{k}) \in \tilde{U}_j \mid \tilde{\mu} \equiv \mu \text{ and } \tilde{k} \equiv k \right\}
\] (19)

Let
\[
O : M \times K \rightarrow \{1, \ldots, \text{card}(M) \cdot \text{card}(K)\}
\]
be a numbering of $U_{eq}^j$ that reflects the priority in the choice of the control action. Namely
\[
\begin{align*}
&\{k^{(1)} > k^{(2)}\} \Rightarrow \left\{O(\mu^{(1)}, k^{(1)}) > O(\mu^{(2)}, k^{(2)})\right\} \\
&\{k^{(1)} = k^{(2)} \text{ and } |\mu^{(1)} - \mu_0| > |\mu^{(2)} - \mu_0|\} \Rightarrow \left\{O(\mu^{(1)}, k^{(1)}) > O(\mu^{(2)}, k^{(2)})\right\}
\end{align*}
\]
where $\mu_0$ is a reference value reflecting the nominal capacitors configuration ($\mu_0 = 1$ is used in the simulations hereafter). This order relation reflects the concern that changes in the capacitors have to be preferred rather than load shedding if the control $n$ is unable to lonely recover the voltage collapse.

We shall denote by $X(t; x_0, \tilde{u})$ the solution of the system’s equations starting from the initial conditions $(0, x_0)$ and under the control sequence $\tilde{u}$. Using this notation, the following two sets are defined
\[
S_{nc}(x_0, j) := \left\{ (\mu, k) \in M \times K \mid \exists \tilde{u} \in \tilde{U}^{(\mu, k)}_j : X(\cdot, x_0, \tilde{u}) \text{ is defined over } [0, N_p \tau] \right\}
\] (20)
in other words, $S_{nc}(x_0, j)$ is the set of pairs $(\mu, k)$ for which there is a control sequence belonging to the class $\tilde{U}^{(\mu, k)}_j$ such that starting from $x_0$, no voltage collapse occurs on the prediction time interval $[0, N_p \tau]$.

Finally, the following subset of $S_{nc}(x_0, j)$ needs to be defined
\[
S_\varepsilon(x_0, j) := \left\{ (\mu, k) \in S_{nc}(x_0, j) \mid \exists \tilde{u} \in \tilde{U}^{(\mu, k)}_j : \forall t \in [0, N_p \tau] \ y(t, x_0, \tilde{u}) \in [1 - \varepsilon, 1 + \varepsilon] \right\}
\] (21)
This is the subset of $S_{nc}(x_0, j)$ (if any) for which the resulting voltage meets the precision requirement over the whole prediction interval.
3.3 The feedback definition

Having at hand the notations of the preceding section, the proposed nonlinear predictive control may be properly defined. Let the pair \((\mu^*(x,j), k^*(x,j))\) ∈ \(M \times K\) be given by

\[
(\mu^*(x,j), k^*(x,j)) := \begin{cases} 
\min_{(\mu,k) \in S_\varepsilon(x,j)} O(\mu,k) & \text{if } S_\varepsilon(x,j) \neq \emptyset \\
\min_{(\mu,k) \in S_{nc}(x,j)} O(\mu,k) & \text{if } S_{nc}(x,j) = \emptyset
\end{cases}
\]

(22)

Note that since \(k_{max}\) is taken equal to 1, \(S_{nc}(x,j)\) is never empty. Again, if \(k > 0.15\) is needed to recover the voltage, then, either this limit is fictitious and higher values are anyway chosen, or this is a real limitation and in this case, the proposed feedback (or any other feedback ?) fails to recover the voltage since collapse occur before \(N_\mu \tau\).

Based on \((\mu^*(x,j), k^*(x,j))\), the ”optimal open-loop” control profile at instant \(j\tau\) is obtained by minimizing the following cost function on the equivalence class \(\tilde{U}^j_{\mu^*(x(j),j),k^*(x(j),j)}\)

\[
\hat{u}(x(j),j) := \min_{\tilde{u} \in \tilde{U}^j_{\mu^*(x(j),j),k^*(x(j),j)}} J(\tilde{u}, x(j),j) := \int_0^{N_p \tau} |y(\tau,x(j),\tilde{u}) - 1|^2 d\tau
\]

(23)

\[
= (\hat{u}_0(x(j), n(j-1)) \ldots \hat{u}_{N_p-1}(x(j), n(j-1))) \in \tilde{U}_j
\]

(24)

yielding the following state feedback law

\[u(t) = \hat{u}_0(x(j), n(j-1)) \quad \forall t \in [j\tau, (j+1)\tau)
\]

(25)

in accordance with the predictive control principle. Note that the dependence on \(n(j-1)\) in (24) comes from the fact that the admissible set \(\tilde{U}^j_{\mu^*(x(j),j),k^*(x(j),j)}\) does depend on \(n(j-1)\) through the index \(j\) that has been used notations simplicity.

4 Some implementation issues

In this section, some details on the implementation of the above feedback control are presented. This is made necessary by the potentially explosive nature of the underlying computation. Indeed, for candidate profiles \(\tilde{u}\) leading to voltage collapse, the solution of the ordinary differential equation modelling the system dynamics may becomes quite long since the solver tries to reduce the integration steps to meet the precision requirement while the corresponding system’s trajectory tends to some singular manifold.

This may needlessly lengthen the computation time so far as to make the whole scheme incompatible with the available computation time. This is particularly true when the original problem will be considered later.

To understand the proposed implementation ”trick”, recall that, as long as no collapse arises, the system equations (13)-(14) are in fact given by

\[
\dot{x} = f(x, \hat{y}(x,u)) \quad ; \quad \hat{y}(x,u) = \sqrt{\hat{w}(x,u)}
\]

(26)

where \(\hat{w}(x,u)\) is a non-negative solution of

\[
\alpha_2(x,u)w^2 + \alpha_1(x,u)w + \alpha_0(x,u) = 0
\]

(27)

that can be proved to admit a non-negative solution if and only if

\[
\Delta(x,u) = \alpha_1^2(x,u) - 4\alpha_0(x,u)\alpha_2(x,u) \geq 0
\]

(28)
The trick is to apply the nonlinear predictive control scheme depicted in the preceding section on the modified dynamical system given by

\[ \dot{x} = f(x, \hat{y}^*(x, u)) \quad \text{where} \quad \hat{y}^*(x, u) = \begin{cases} \sqrt{\hat{u}(x, u)} & \text{if} \quad \Delta(x, u) \geq 0 \\ 0 & \text{if} \quad \Delta(x, u) < 0 \end{cases} \]  

(29)

namely, the true dynamic is embedded in a fictitious one that is identical on the admissible domain and is still ”computable” on the forbidden one.

Using the modified system (29), the admissible set definition is re-written as follows

\[ S_{nc}(x_0, j) := \{ (\mu, k) \in M \times K \mid \exists \tilde{u} \in \tilde{U}^{(\mu,k)}_j : y(\cdot, x_0, \tilde{u}) \neq 0 \ \text{over} \ [0, N_p] \} \]  

(30)

Remark 3

It is worth noting that the control scheme proposed above uses the assumption that the value of the reactance X is known. This may imply the use of an estimation scheme or an observer for X. In both cases, an estimated value \( \hat{X}(j) \) is used in the computation of the predicted cost associated to some candidate profile. The design of such an estimation (observer) scheme (that is necessary whatever is the control scheme being used) is not addressed here and must be the object of a separate study.

5 Some validating simulations

In this section, some simulations are proposed in order to assess the validity of the proposed predictive control scheme. Each group of simulations enables a particular feature to be underlined.

✓ Figure 4 shows an ”easy situation” in the sense that there is no need to introduce capacitor or load shedding in order to avoid voltage collapse. This is because the stationary value of \( n \) is close to the initial value \( n(0) \).

✓ Figures 5-6 show more difficult situation since the initial values of \( n(0) \) are respectively 0.9 and 0.8. This necessitates the introduction of capacitors in order to recover the voltage and avoid collapse. However, this is only necessary in the transient phase. Note that the resulting closed-loop behaviour of the control is more rich than the open-loop parameterization. This is a key feature in predictive control scheme.

✓ Figures 7-9 show what happens when increasing value of the inductance \( X \) is taken. This corresponds to increasing difficulty leading to a transient load shedding for the value \( X = 0.48 \).

✓ Figures 9-11 show the role of the precision parameter \( \varepsilon \). For, several values of \( \varepsilon = 0.05, 0.1 \) and 0.15 are used. It comes out that for greater values of \( \varepsilon \), the precision on \( y \) is getting worse. Note that for higher values of \( \varepsilon \) enables the mean value of the capacitor to be reduced at the price of bas precision.

✓ To show what happens in the absence of the precision constraint, figure 12 shows simulation with \( \varepsilon = \infty \). In other words, only the voltage collapse avoidance is used in the constrained predictive control. Note the poor regulation performance and the oscillation due to the relative permissivity of the corresponding constraint.

✓ Finally, figures 13-14 show the influence of the prediction horizon length. Greater prediction horizons reduce the frequency of the oscillations.

Note generally that if oscillations are to be reduced, this must be explicitly expressed in the cost function or in the constraint. At the time being, nothing is explicitly done to avoid them and the predictive control just meet the specification requirements.
Figure 4: $X = 0.2$, $n(0) = 0.98$, $N_p = 20$ and $\varepsilon = 0.05$

Figure 5: $X = 0.2$, $n(0) = 0.9$, $N_p = 20$ and $\varepsilon = 0.05$

Figure 6: $X = 0.2$, $n(0) = 0.8$, $N_p = 20$ and $\varepsilon = 0.05$
Figure 7: $X = 0.4$, $n(0) = 0.8$, $N_p = 20$ and $\varepsilon = 0.05$

Figure 8: $X = 0.45$, $n(0) = 0.8$, $N_p = 20$ and $\varepsilon = 0.05$

Figure 9: $X = 0.48$, $n(0) = 0.8$, $N_p = 20$ and $\varepsilon = 0.05$
Figure 10: $X = 0.48, n(0) = 0.8, N_p = 20$ and $\varepsilon = 0.1$

Figure 11: $X = 0.48, n(0) = 0.8, N_p = 20$ and $\varepsilon = 0.15$

Figure 12: $X = 0.48, n(0) = 0.8, N_p = 20$ and $\varepsilon = \infty$
Figure 13: $X = 0.48, n(0) = 0.8, N_p = 40$ and $\varepsilon = 0.05$

Figure 14: $X = 0.48, n(0) = 0.8, N_p = 80$ and $\varepsilon = 0.05$
References

