Survey of observability and identification for hybrid systems

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Outline

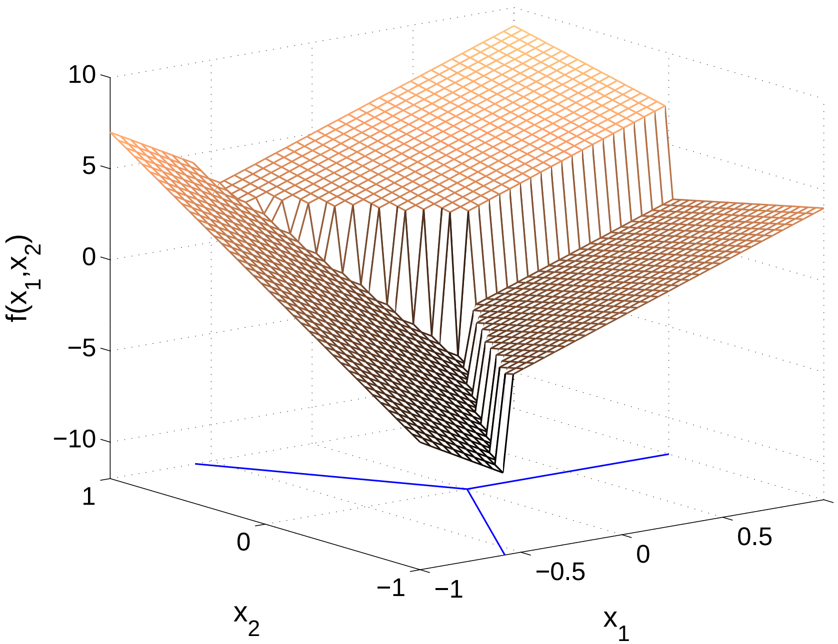
- Two estimation problems for hybrid systems given input/output data
  - ✓ Identification of hybrid models
  - ✓ Filtering of the hybrid state
- Identification of hybrid systems
  - ✓ Statement of the problem
  - ✓ A Mixed-Integer Programming procedure
  - ✓ A Bounded-Error procedure
- Observability of hybrid systems
  - ✓ Conditions for observability (in infinitesimal time, single-event)
  - ✓ Conditions for Generic Final-State Asymptotical Determinability
  - ✓ Undecidability
- Observer design

Piecewise Affine Systems (1/2)

PWA systems form a special class of nonlinear systems whose state and output maps are both piecewise affine

A PWA map $f : \mathcal{X} \to \mathbb{R}^q$ is defined as follows:

$$f(x) = \begin{cases} \theta_1' \varphi & \text{if} \quad x \in \mathcal{X}_1 \\ \vdots & \qquad \vdots \\ \theta_s' \varphi & \text{if} \quad x \in \mathcal{X}_s \end{cases} \qquad \varphi = \begin{bmatrix} x \\ 1 \end{bmatrix}$$



where:

- $\mathcal{X}_i$ are convex polyhedra
- $\bigcup_{i=1}^{s} \mathcal{X}_i = \mathcal{X} \subseteq \mathbb{R}^p$
- $\mathcal{X}_i \bigcap \mathcal{X}_j = \emptyset \; \forall \, i \neq j$

Piecewise Affine Systems (2/2)

- State Space Form

$$\begin{cases} \zeta_{k+1} = A_i\,\zeta_k + B_i\,u_k + b_i + v_k \\ y_k = C_i\,\zeta_k + D_i\,u_k + d_i + w_k \end{cases} \quad \text{if} \begin{bmatrix} \zeta_k \\ u_k \end{bmatrix} \in \Omega_i$$

- Regression Form

$$y_k = \theta_i' \varphi_k + \eta_k \quad \text{if } x_k \in \mathcal{X}_i\,, \quad \text{where } \varphi_k = \begin{bmatrix} x_k \\ 1 \end{bmatrix}$$

A PWA system in regression form for which:

$$x_k = [y_{k-1} \ldots y_{k-n_a}\; u_{k-1} \ldots u_{k-n_b}]'$$

is called *PieceWise affine Autoregressive Exogenous* (PWARX) system

Motivations for PWA Identification

- PWA maps have universal approximation properties
  (Lin and Unbehauen, 1992; Breiman, 1993)
- PWA systems are equivalent to several classes of hybrid systems
  (Bemporad et al., 2000; Heemels et al., 2001)
- PWA models are suitable for analysis and control of classes
  of nonlinear systems (e.g., Chua et al., 1982)

Hence:

- PWA systems form a nonlinear black-box structure

  "*A model structure that is prepared to describe virtually any nonlinear dynamics*" (Sjöberg et al., 1995)
- PWA Identification techniques can be applied to obtain hybrid models

The PWA Identification Problem

Consider PWARX models in the form: $y_k = f(x_k) + e_k$

- ✓ $y_k \in \mathbb{R}$ and $u_k \in \mathbb{R}$ are the *system output* and *input*, respectively
- ✓ $x_k = [y_{k-1} \dots y_{k-n_a}\ u_{k-1} \dots u_{k-n_b}]' \in \mathcal{X}$ is the *regression vector*
- ✓ $e_k \in \mathbb{R}$ is the *prediction error*
- ✓ $f(\cdot)$ is a *PWA map*: $f(x) = \theta_i' \varphi$ if $x \in \mathcal{X}_i$, $\varphi = \begin{bmatrix} x \\ 1 \end{bmatrix}$
- ✓ $\{\mathcal{X}_i\}_{i=1}^s$ is a *polyhedral partition* of the regressor set $\mathcal{X} \subseteq \mathbb{R}^n$

Given N data points (y_k, x_k), $k = 1, \dots, N$, find the PWARX model that best matches the given data according to the specified criterion of fit

$\Rightarrow$ Involves the estimation of s, $\{\theta_i\}_{i=1}^s$, and $\{\mathcal{X}_i\}_{i=1}^s$

Approaches to identification of PWA models

- A wide literature considering *continuous* PWA maps (*e.g.*, HH functions)
- Recent approaches allow for *discontinuous* PWA maps, *e.g.*:
  - ✓ K-means clustering-based procedure
    `(Ferrari-Trecate et al., Automatica, 2003)`
  - ✓ Adapted weights procedure
    `(Ragot et al., CDC 2003)`
  - ✓ Bayesian procedure
    `(Juloski et al., CDC 2004)`
  - ↪ require to fix a priori the number of submodels
- Other approaches deal with switched ARX systems (no partition), *e.g.*:
  - ✓ Algebraic procedure
    `(Vidal et al., CDC 2003)`

A Mixed-Integer Programming approach (1/2)

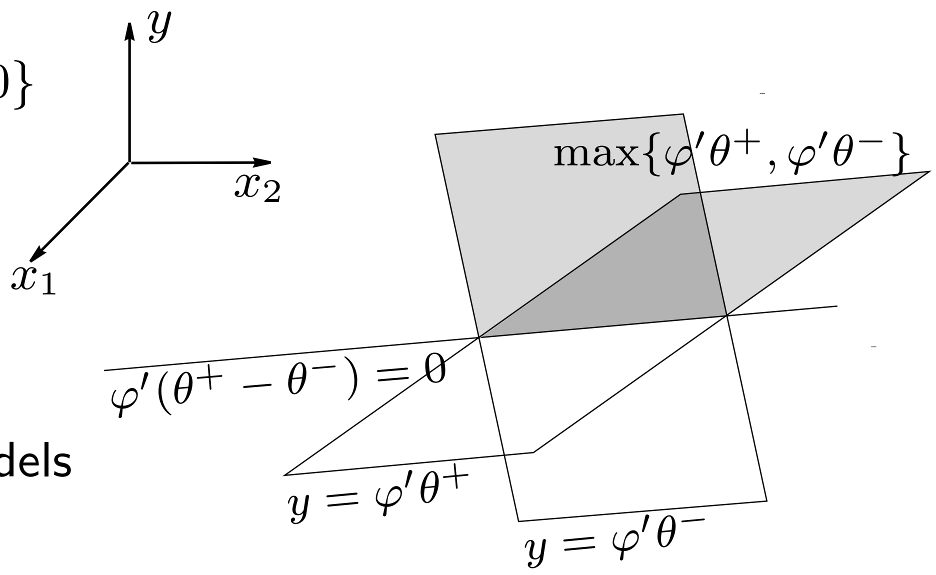
(Roll, Bemporad and Ljung, Automatica, 2004)

Optimal identification of Hinging-Hyperplane ARX (HHARX) models:

$$y(k) = f(x(k); \theta) + e(k; \theta)$$

where:

- ✓ $f(x(k); \theta) = \varphi(k)'\theta_0 + \sum_{i=1}^{M} \sigma_i \max\{\varphi(k)'\theta_i, 0\}$
- ✓ $\varphi(k) = [x(k)' \ 1]'$ and $\theta = [\theta_0' \ \theta_1' \ldots \theta_M']'$
- ✓ $\sigma_i \in \{-1, 1\}$ are fixed a priori



Note that:

- HHARX models form a subclass of PWARX models for which the PWA map $f(\cdot)$ is continuous
- The number of submodels s is bounded by the quantity $\sum_{j=0}^{n} \binom{M}{j}$

A Mixed-Integer Programming approach (2/2)

(Roll, Bemporad and Ljung, Automatica, 2004)

The optimal parameter vector θ^* is selected by solving (for $p = 1$ or 2):

$$\theta^* = \arg\min_{\theta} \sum_{k=1}^{N} \left| y(k) - f(x(k); \theta) \right|^p$$

- Prediction error method
- Reformulated as a mixed-integer linear or quadratic program (MILP/MIQP) by introducing binary variables
- Optimality at the cost of a theoretically very high worst-case computational complexity
- Mainly suitable for small-scale problems
- Extensions are also possible for handling non-fixed σ_i, discontinuities, general PWARX models, etc.

A bounded-error approach (1/2)

(Bemporad, Garulli, Paoletti and Vicino, HSCC 2003 - CDC 2004)

The identified model is required to satisfy the following *bounded-error condition*:

$$\left| y_k - f(x_k) \right| \leq \delta \,, \quad \forall\, k = 1, \ldots, N$$

i.e., the prediction error $e_k = y_k - f(x_k)$ must be bounded by a given quantity $\delta > 0$ for all the samples in the estimation data set

- The bound δ determines both the model accuracy and the number of submodels s
- Reformulation of the identification problem:

> Given N data points (y_k, x_k), $k = 1, \ldots, N$, estimate the *minimum* integer s, parameter vectors $\{\theta_i\}_{i=1}^{s}$, and regions $\{\mathcal{X}_i\}_{i=1}^{s}$ such that the identified PWARX model satisfies the bounded error condition

A bounded-error approach (2/2)

(Bemporad, Garulli, Paoletti and Vicino, HSCC 2003 - CDC 2004)

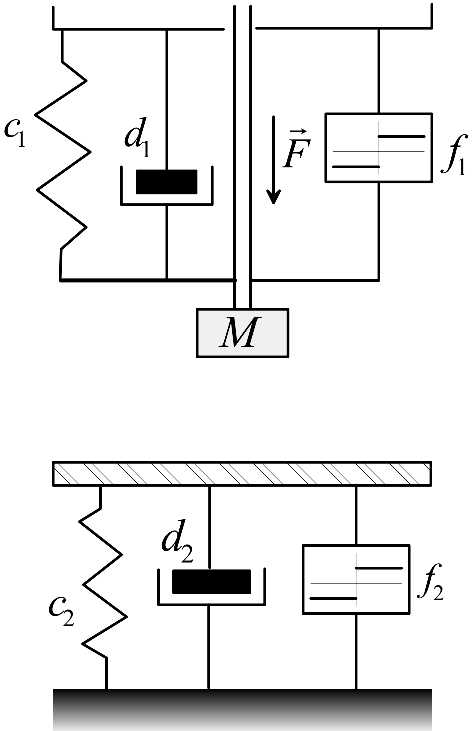
A two-stage procedure is proposed to solve the identification problem:

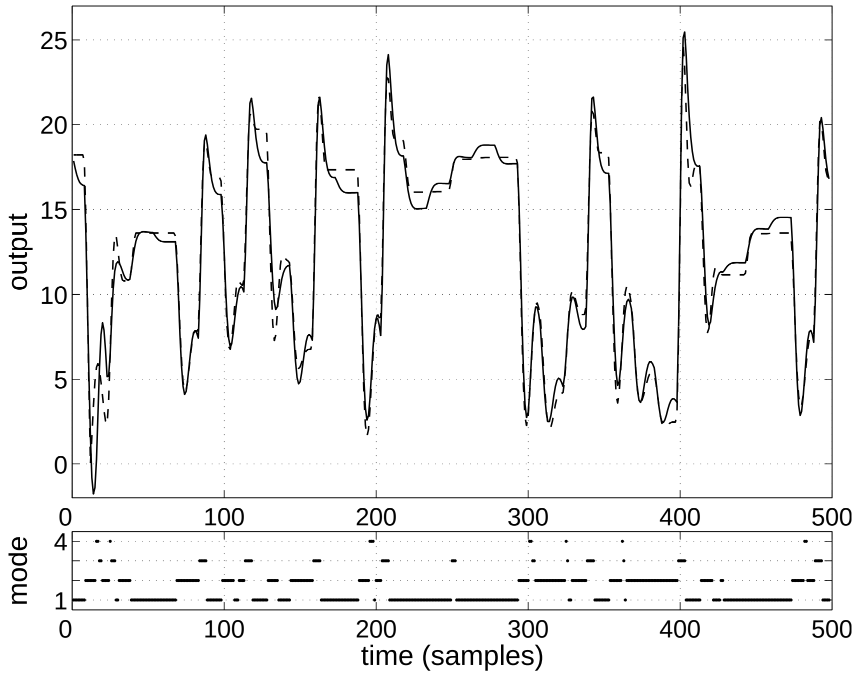
1. Estimation of s and $\{\theta_i\}_{i=1}^{s}$
   - ✓ *Initialization*: partition an infeasible system of linear inequalities into a minimum number of feasible subsystems
     (NP-hard problem $\Rightarrow$ suboptimal greedy method is used)
   - ✓ *Refinement*: improves classification by alternating between data point reassignment and parameter update
2. Estimation of $\{\mathcal{X}_i\}_{i=1}^{s}$
   - ✓ Separate the clusters of data by exploiting two-class or multi-class linear separation techniques

A case study: Electronic component placement process (1/2)

Input: Force F applied to the mounting head M

Output: Position of the mounting head M (dashed: measured; solid: simulated)

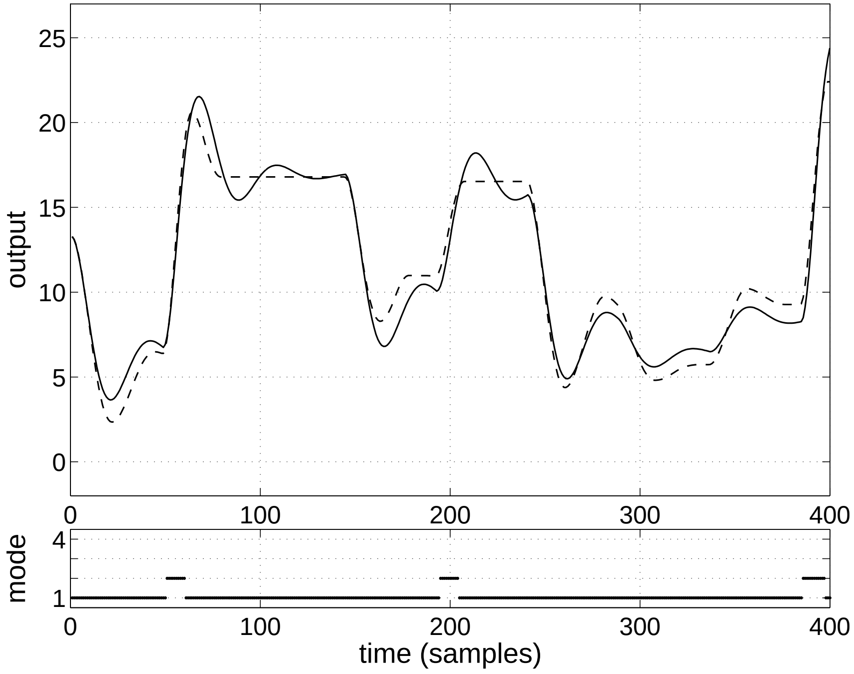


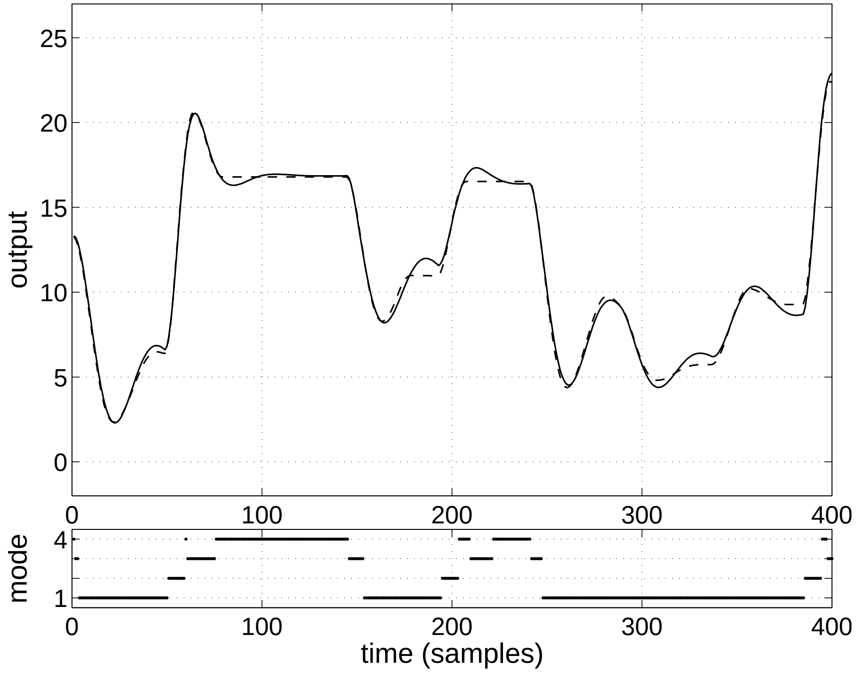


MIP procedure, $N = 150$, $M = 2$

FIT$= 81.55\%$

A case study: Electronic component placement process (2/2)





BE procedure, $N = 1000$, $s = 2$ $(\delta = 0.06)$
FIT$= 81.32\%$

BE procedure, $N = 1000$, $s = 4$ $(\delta = 0.04)$
FIT$= 93.48\%$

This application shows that the bound δ can effectively be used as a tuning parameter for trading off between model complexity and model accuracy

Observability of hybrid systems

Motivation: can we estimate the states of a hybrid system from a certain set of input/output measurements?

- Many different notions of *observability* exist
- Related to the degree to which the state can be determined, *e.g.*,
  - ✓ (initial-state) observability
  - ✓ current-state (or final-state) observability (also known as *reconstructability*)
- Related to the observation time needed for state reconstruction
  - ✓ observability in infinitesimal time
  - ✓ observability in finite time
  - ✓ observability in infinite time

Sufficient conditions for observability of PWA systems (1/2)

(Collins and van Schuppen, HSCC 2004)

Consider the autonomous PWA system:

$$\begin{aligned}\dot{x}(t) &= A_q x(t) + a_q,\ \ x(t_0) = x_0\\ y(t) &= C_q x(t) + c_q\end{aligned}$$

- $q \in \{1, \dots, s\}$ is the discrete-state, and $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^p$ are the continuous state and output, respectively
- Guards and continuous-state transitions are defined by affine equations and functions
- An event e occurring at (q, x) is called detectable at (q, x) if it produces a measurable change in the output
- The system is called event detectable if all events are detectable at all states
  $\Rightarrow$ allows only determination of the time at which an event occurs
- The system is called observable if the mapping from the initial state to the output trajectory is injective

Sufficient conditions for observability of PWA systems (2/2)

(Collins and van Schuppen, HSCC 2004)

Theorem. A PWA system is observable if:

1. all events are detectable;
2. for all timed-event sequences $\{t_i, i \in \mathbb{Z}_+\}$, there exists at most one state (q_0, x_0) which is a solution of the trajectory equations for any possible event sequence.

Remarks

- Dependence on timed-event sequence is practically untractable
- Checkable (sufficient) conditions are derived for *observability in infinitesimal time* and *single-event observability*

Observability of PWA systems - Undecidability

(Collins and van Schuppen, CDC 2004)

Consider the autonomous *rational* PWA system:

$$\begin{aligned} \dot{x}(t) &= f(x(t)), \quad x(t_0) = x_0 \\ y(t) &= h(x(t)) \end{aligned}$$

✓ $f : \mathcal{X} \to \mathcal{X}$ and $h : \mathcal{X} \to \mathcal{Y}$ are PWA functions with *rational* coefficients

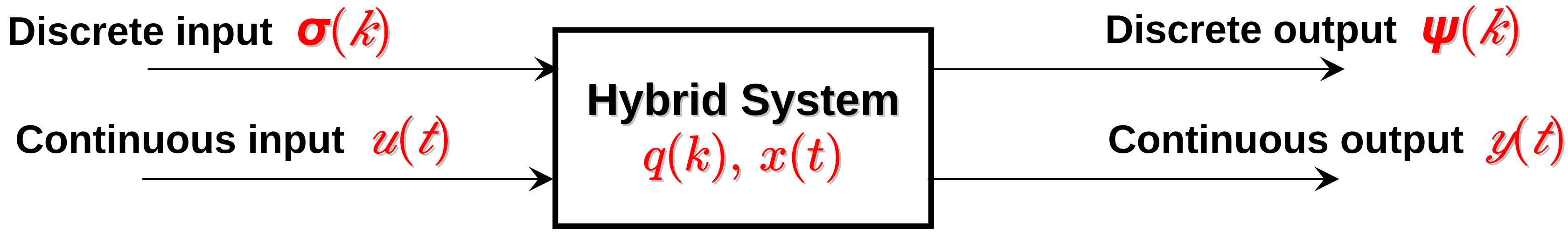
✓ $\mathcal{X} = \cup_{i=1}^{s} \mathcal{X}_i$, and $\mathcal{X}_i$ are specified by linear inequalities with *rational* coefficients

Theorem. Observability is undecidable for the above class of systems with finitely many possible initial states.

↪ Argument of the proof. Deciding observability is equivalent to the *halting problem* for a Turing machine

Theorem. Discrete-state observability is undecidable for discontinuous PWA systems.

Hybrid systems formalism



$$\mathcal{H} = (Q, \Sigma, \Psi, \varphi, \phi, \eta, X, U, \Omega, Y, f, h, r)$$

Discrete-state dynamics

$$\begin{aligned} q(k+1) &\in \varphi\left(q(k), \sigma(k+1)\right) \\ \sigma(k+1) &\in \phi\left(q(k), x(t_{k+1}^{-}), u(t_{k+1}^{-})\right) \\ \psi(k+1) &\in \eta\left(q(k), \sigma(k+1), q(k+1)\right) \end{aligned}$$

Continuous-state dynamics

$$\begin{aligned} \dot{x}(t) &= f(q_i, x(t), u(t), w(t)) = A_i\ x(t) + B_i\ u(t) + w(t) \\ y(t) &= h(q_i, x(t)) = C_i\ x(t) \end{aligned}$$

Resets

$$x(t_k) = x(t_k^{+}) = r(q_i, q_j, x(t_k^{-})) = R_{ij}^{1}\ x(t_k^{-}) + R_{ij}^{0}$$

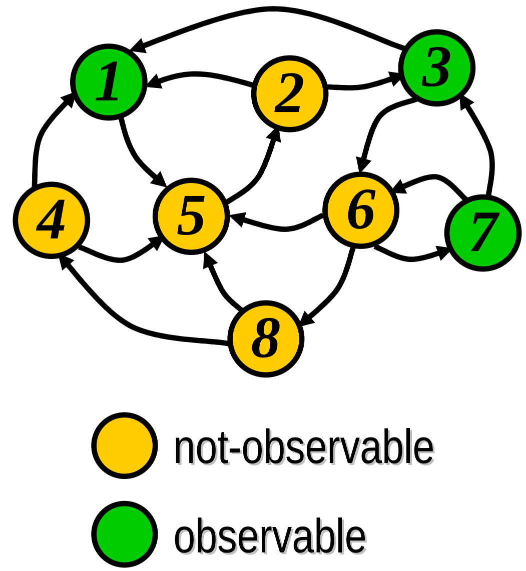
Generic final-state asymptotic determinability

- A hybrid system is *Generic Final-State Asymptotically Determinable (GFSAD)* if any generic input/output experiment permits:
  - √ the identification of the discrete state after a finite number of transitions, and
  - √ the asymptotic determination of the continuous state
- A hybrid system is *Current Location Observable (CLO)* if the discrete state can be determined from observations of the discrete output only for any initial discrete state and any discrete input sequence
- The class of hybrid systems considered is that of *living hybrid systems with no-multiple transitions*
  - √ *admit only executions that are non-Zeno and have an infinite number of transitions separated by continuous evolutions*

Sufficient conditions of GFSAD

(Balluchi et al., CDC 2003 - MTNS 2004)

Sufficient conditions of GFSAD have been derived for all combinations of *continuous domain* and *discrete domain* observability properties



Continuous domain properties

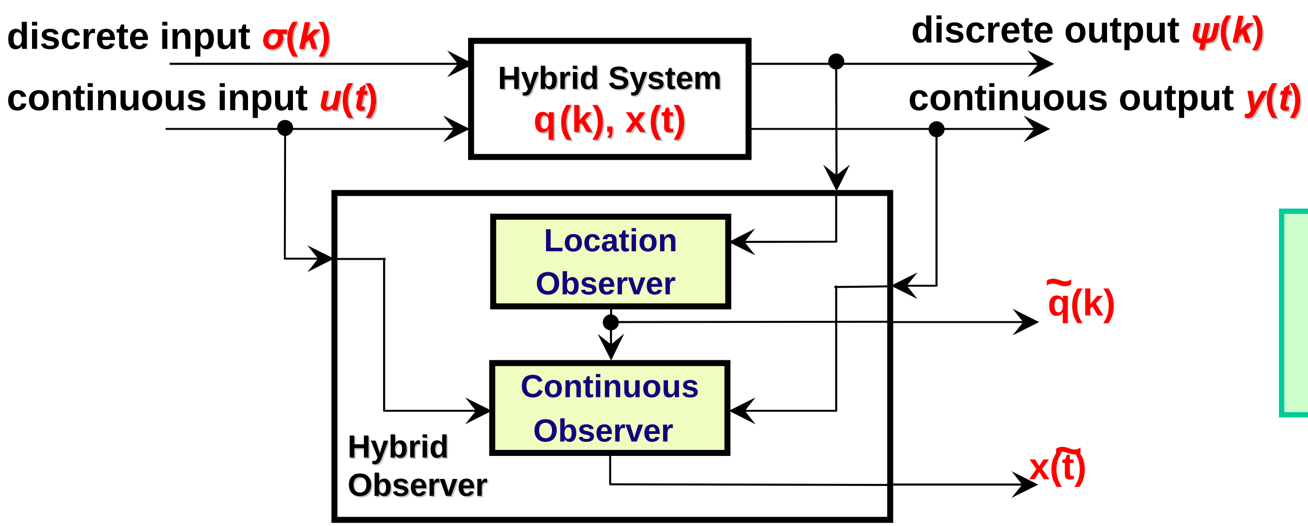
- Observable subsystems
- At least one observable subsystem in each cycle
- No overlapping between cycles composed by not-observable subsystems
- Some overlapping between cycles composed by not-observable subsystems

Discrete domain properties

- Current-location observable system
- Not current-location observable system

Final-state asymptotic determination (1/2)

(Balluchi et al., HSCC 2002 - MED 2002)

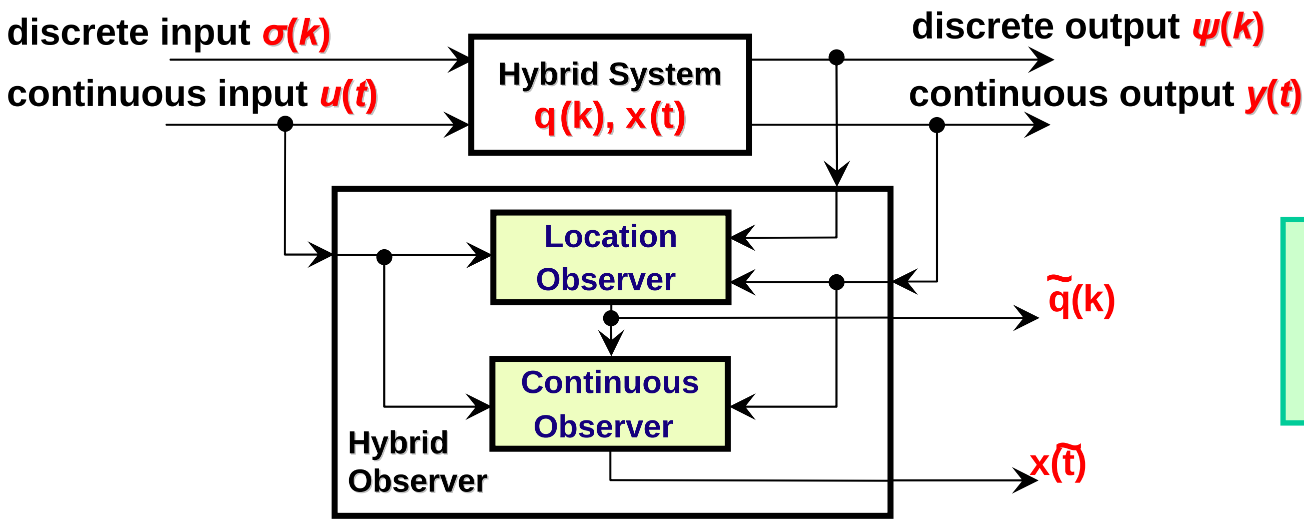


Current Location Observable

- The *Location Observer* is obtained by computing the current-location observation tree of the hybrid system FSM
- The *Continuous Observer* is a switched Luenberger observer with resets

Final-state asymptotic determination (2/2)

(Balluchi et al., HSCC 2002 - MED 2002)



Not Current Location Observable

- The *Location Observer* now incorporates a *signature generator* inspired by failure detection and identification techniques

Conclusions

- A review of results on identification and observability of hybrid systems achieved in the European Project CC (Computation and Control)
- Many open issues...
- ...in identification, *e.g.,*
  - √ *incorporate prior knowledge*
  - √ *different model orders in different modes*
  - √ *estimation of the partition (nonconvex and non-connected regions)*
- *...in observability, e.g.,*
  - √ *decidability*
  - √ *theory of realisation*