Controlling Hybrid Systems

From Theory to Application

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Lessons learned from a decade of Hybrid System Research

- For (engineering) research on HS to be sustainable we need to impact applications

⇒We need to focus on problems that are critical for applications

- Our new tools need to solve problems that cannot be solved otherwise.

Outline

1. Background
   - Multi-Parametric Programming
   - Controller Computation
2. Low Complexity Controllers
   - The Three Levers of Complexity
   - Minimum-Time Controller
   - *N*-Step Controller
3. Industrial Applications
   - Control of a DC-DC Converter
   - Direct Torque Control
4. Conclusions

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Parametric Programming Definition

$$\begin{aligned} J^*(x) &= \min_z f(z,x) \\ \text{subj. to } & g(z,x) \le 0 \end{aligned}$$

z: Decision Variables x: Parameters

Goal Determine optimal cost $J^*(x)$ and optimizer $z^*(x)$ for a range X^* of parameters.

Terminology Sensitivity analysis, multi-parametric (mp) programming; depending on problem: mpLP, mpQP, mpMILP, mpMIQP, etc.

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mp-QP

$$\begin{aligned} J^*(z) &= \min_z \tfrac{1}{2} z' H z \\ \text{subj. to } & Gz \le W + Sx \end{aligned}$$

$z \in \mathbb{R}^s, \quad G \in \mathbb{R}^{c \times s}, \quad W \in \mathbb{R}^c, \quad S \in \mathbb{R}^{c \times n}$

Definitions:

- Feasible Set X^*
  For each $x \in X^*$ there exists an optimizer $z^*(x)$ such that the constraints ($Gz^*(x) \le W + Sx$) are satisfied.
- Value Function $J^*(x)$, $x \in X^*$
- Optimizer $z^*(x)$, $x \in X^*$

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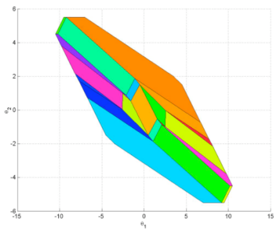
Characteristics of mp-QP Solution

- The optimizer $z^*(x)$ is continuous and piecewise affine.
- The feasible set X^* is convex and partitioned into polyhedral regions.
- The value function $J^*(x)$ is convex, piecewise quadratic and C^1.

$$z^*(x) = \left\{ \begin{array}{lll} F_1 x + G_1 & \text{if} & H_1 x \le K_1 \\ \vdots & \vdots & \vdots \\ F_R x + G_R & \text{if} & H_R x \le K_R \end{array} \right\}$$



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Control of Constrained PWA Systems

System

- Discrete ***PWA*** Dynamics $x(k+1) = f_{\text{PWA}}(x(k), u(k))$
- Constraints on the state $x(k) \in \mathcal{X}$
- Constraints on the input $u(k) \in \mathcal{U}$

$$\left.\begin{matrix} x(k) \in \mathcal{X} \\ u(k) \in \mathcal{U} \end{matrix}\right\} C^x x(k) + C^u u(k) \leq C^0$$

Objectives

- Stability (feedback is stabilizing)
- Feasibility (feedback exists for all time)
- Optimal Performance

Constrained Finite Time Optimal Control (CFTOC) of PWA Systems

Linear Performance Index (p=1,∞)

$$J^*(x) := \min_U \|Px_T\|_p + \sum_{k=0}^{T-1} \|Qx_k\|_p + \|Ru_k\|_p$$

Constraints

$$\begin{cases} x_0 = x, \\ x_{k+1} = f_{\text{PWA}}(x_k, u_k), \\ C^x x_k + C^u u_k \leq C^0 \end{cases}$$

Algebraic manipulation ➡ Mixed Integer Linear Program (MILP)

$U^*(x) = \{u_0^*, u_1^*, \ldots, u_{T-1}^*\}$

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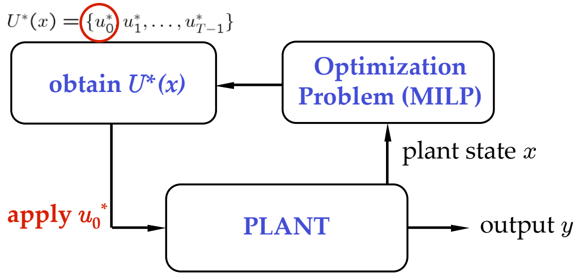
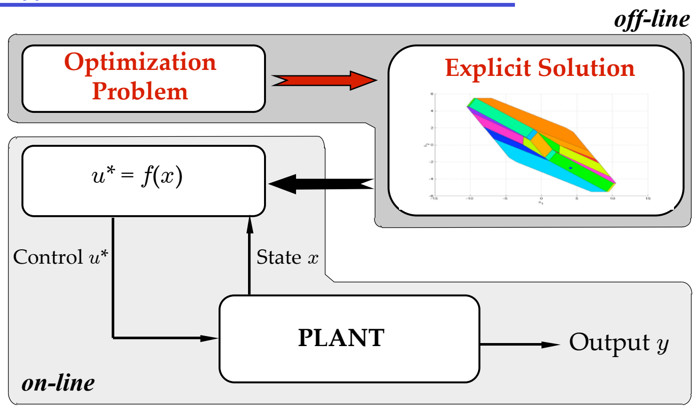
Receding Horizon Control

Receding Horizon Control *On-Line* Optimization



Receding Horizon Policy
Off-Line Optimization



Multi-parametric controllers

Algorithms have been developed for over 5 years:

…Minimization of linear and quadratic objectives
(Baotic, Baric, Bemporad, Borrelli, De Dona, Dua, Goodwin, Grieder, Johansen, Mayne, Morari, Pistikopoulos, Rakovic, Seron, Toendel)

…Minimum-Time controller computation
(Baotic, Grieder, Kvasnica, Mayne, Morari, Schroeder)

…Infinite horizon controller computation
(Baotic, Borrelli, Christophersen, Grieder, Morari, Torrisi)

…Computation of robust controllers
(Borrelli, Bemporad, Kerrigan, Grieder, Maciejowski, Mayne, Morari, Parrilo, Sakizlis)

⇒ Computation schemes are mature !

Multi-parametric controllers

PROs:

– Easy to implement
– Fast on-line evaluation (parallel computation)
– Analysis of closed-loop system possible

CONs:

– Number of controller regions can be large
– Off-line computation time may be prohibitive
– Computation scales badly.

⇒ controller complexity is the crucial issue

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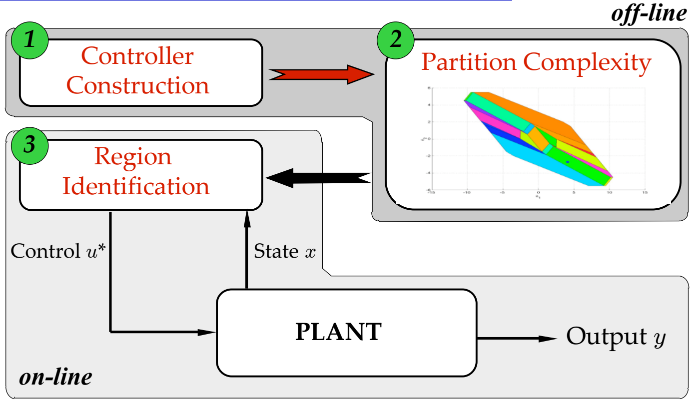
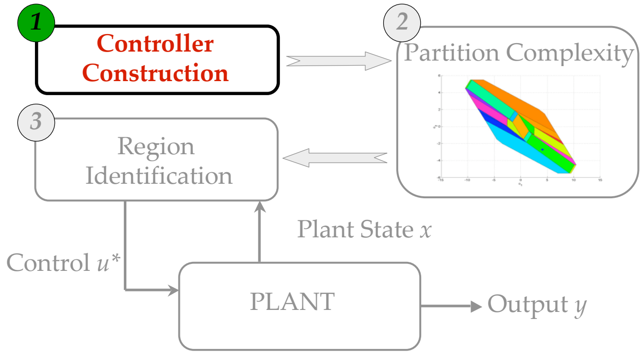
3 Complexity Levers of Receding Horizon Control



1st Lever for Complexity Reduction



1st Lever for Complexity Reduction

Objective

Compute controller partition as quickly as possible.

Why is computation time an issue?

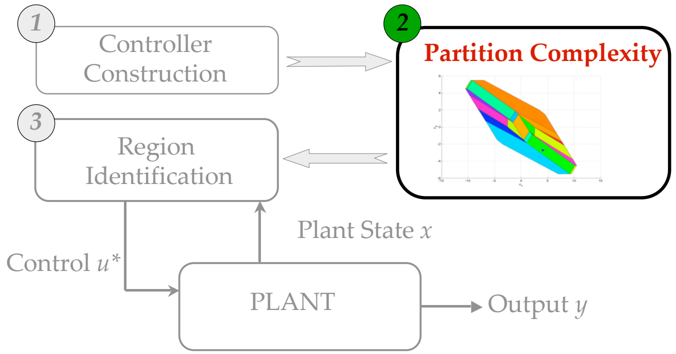
- For LTI systems, controller computation time correlates to controller complexity
- For PWA systems, controller computation time may not correlate to controller complexity

However...

- Controller complexity and runtime grows exponentially with problem size

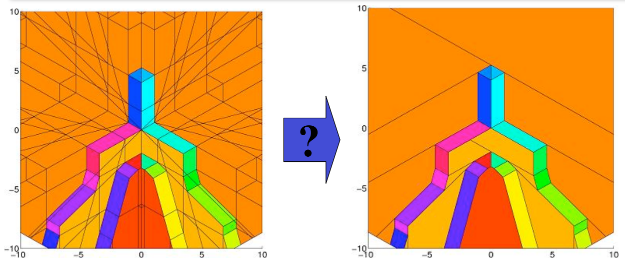
⇒ Controller computation is not a bottleneck

2nd Lever for Complexity Reduction



2nd Lever for Complexity Reduction - Region Merging

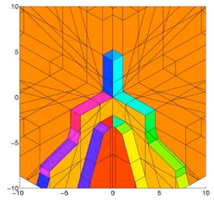
Merge polyhedra with same feedback law such that resulting system is **minimal** in number of controller regions



Two step **algorithm**: 1.) Compute **hyperplane arrangement**
2.) Merge using **Boolean Minimization**

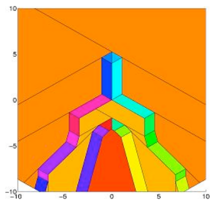
2nd Lever for Complexity Reduction - Region Merging

Optimal merging:
hyperplane arrangmt: 34 sec
merging (boolean min.): 5 sec

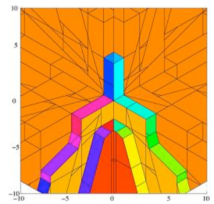


252 polyhedra

Greedy merging:
merging based on LPs: 17 sec



39 polyhedra



189 polyhedra

Control Objective vs. Partition Complexity

Objective

– Compute controller partition with as few regions as possible
– Guarantee stability and constraint satisfaction

Observation

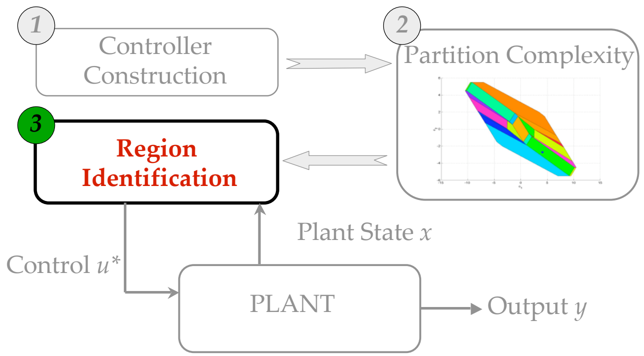
Complex objectives yield complex controllers

Approach

Use simpler objectives to obtain simpler controllers

⇒ **Controller complexity is a bottleneck**

3rd Lever for Complexity Reduction



3rd Lever for Complexity Reduction

Objective

For a given partition, identify controller region as quickly as possible.

Algorithms

Identification of region in time logarithmic in the number of regions is possible *(Bemporad, Grieder, Johansen, Jones, Rakovic, Toendel)*

However…

Scheme is only applicable if controller partition can be obtained

⇒ Region identification is not a bottleneck

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Minimum Time Controller

- Specify "simpler" performance objective:
  - Drive state into target set in minimum-time
  - Instead of solving *one* problem of size *N*, solve *N* problems of size *one*
- Stability and constraint satisfaction are guaranteed

Result: **Fewer Controller Regions**
"Fast" Construction of Control Law

(Grieder, Morari; CDC 2003)
(Grieder, Kvasnica, Baotic, Morari; ACC 2004)

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N-Step Control

- Do not enforce closed-loop stability:
  Solve "standard" constrained finite time optimal control problem
  + additional invariant set constraint on x_1
- Constraint satisfaction and optimal performance are guaranteed
- Analyze stability of resulting closed-loop system

Result: **Significantly Fewer Controller Regions**
"Fast" Construction of Control Law

(Grieder, Parrilo, Morari; CDC 2003)
(Grieder, Kvasnica, Baotic, Morari; to appear in Automatica)

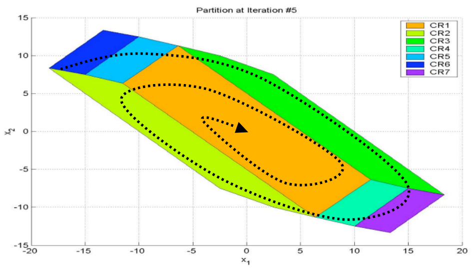
N-Step Controller Algorithm

1. Obtain an invariant controller partition + control law
   (e.g., last partition obtained with minimum-time algorithm).
2. For this partition, solve LMI to find a Lyapunov function for the partition.
3. If such a function is found, stability AND feasibility are guaranteed.

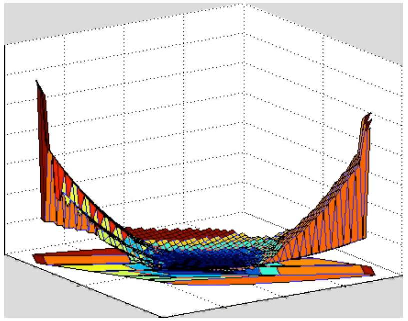
Note: Stability without invariance is useless…

N-Step Controller: Invariant Controller Partition

- Simple invariant PWA controller partition obtained with the iterative algorithm.
- Feasibility is guaranteed for all time.

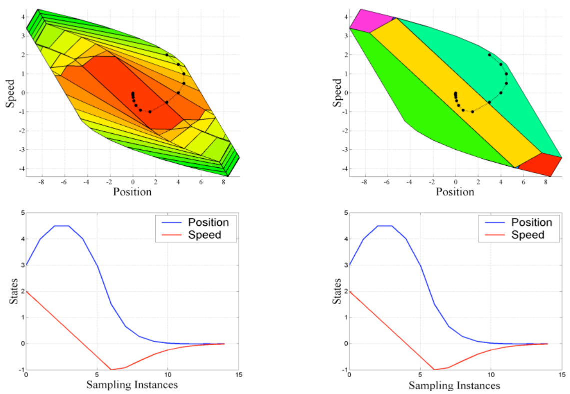


N-Step Controller: Stability Analysis



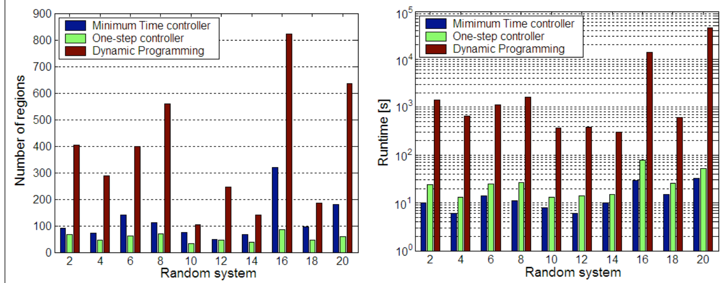
Numerical Example

Optimal Control | *N*-Step Control



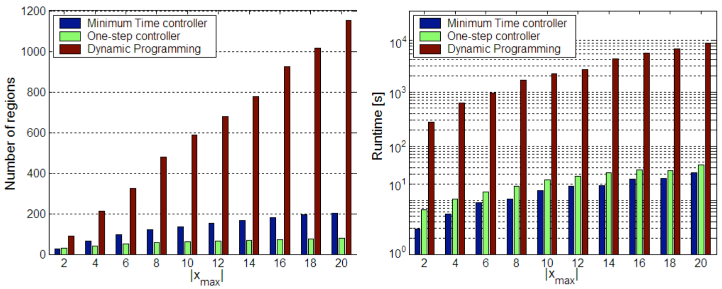
Numerical Examples

Controllers for **20 random PWA systems** with 2 states, 1 input and 4 different dynamics were computed…



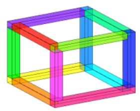
Numerical Examples

Impact of increasing the size of the feasible state space…



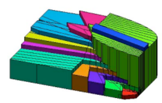
Multi Parametric Toolbox

- All results and plots were obtained with the MPT toolbox



http://control.ethz.ch/~mpt

- MPT is a MATLAB toolbox that provides efficient code for
  - (Non)-Convex Polytope Manipulation
  - Multi-Parametric Programming
  - Control of PWA and LTI systems



MPT in the World

2000+ downloads in 1 year

Rated 4.5 / 5 on `mathworks.com`

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Application Overview

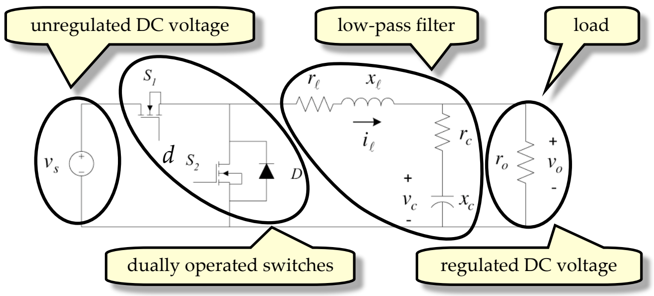
DC-DC (and AC-DC) conversion in

- Power supplies, UPS, battery chargers,…
- DC Motor Drives
- Power Systems (HVDC transmission, …)
- Demanding applications (air and space, …)

Switch-mode DC-DC Converter

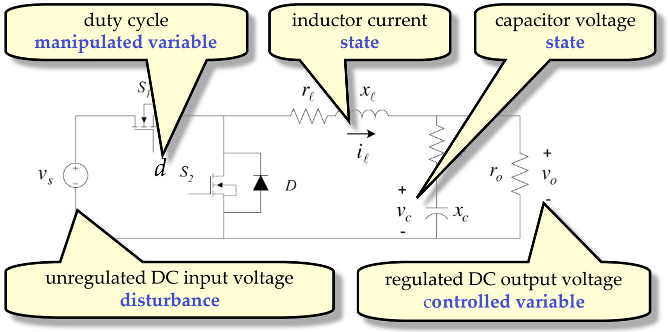
Switched circuit: supplies power to load with constant DC voltage

Illustrating example: **synchronous step-down** DC-DC converter



Control Objective

Regulate DC output voltage by appropriate choice of duty cycle

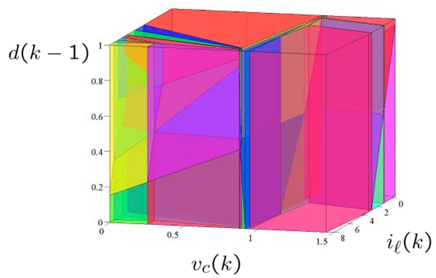


State-feedback Controller: Polyhedral Partition

PWA state-feedback control law:

45 polyhedra

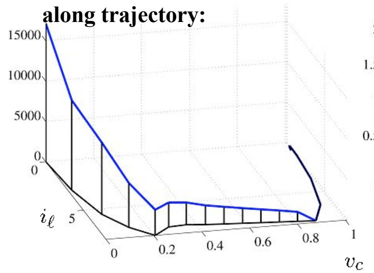
in 10s computation time using the MPT toolbox



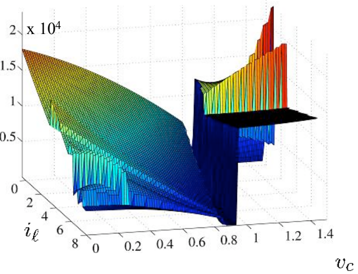
Colors correspond to the 45 polyhedra

State-feedback Controller: Lyapunov Function

Value of Lyapunov Fct. along trajectory:



Value of Lyapunov Fct. for *d*(*k*-1) = 1:

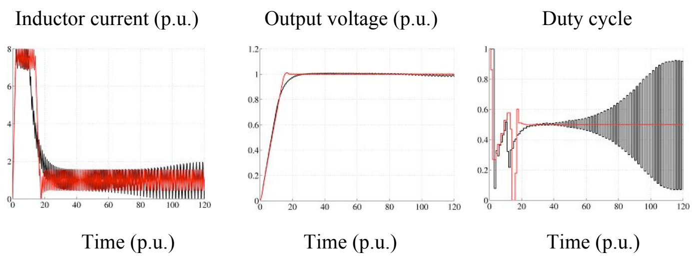


Colors correspond to value of Lyap. Fct.

PWQ Lyapunov function shows **exponential stability**

Case 2: Start-up with Small Input Volt.

x(0) = [0, 0]^T p.u.; v_s = 2.1 (instead of 3) p.u.; r_o=1 p.u.



Industrial controller unstable!

Optimal control — Industrial standard

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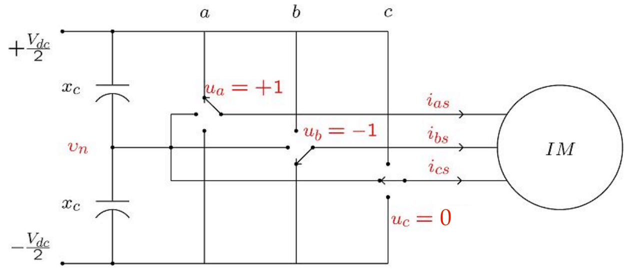
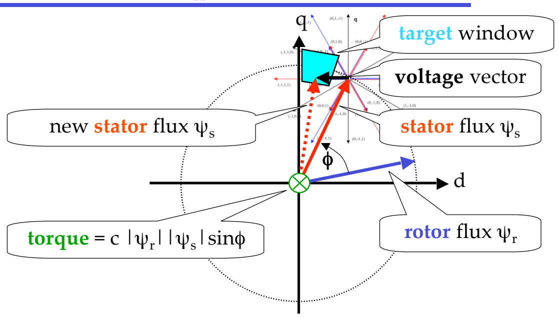
Physical Setup



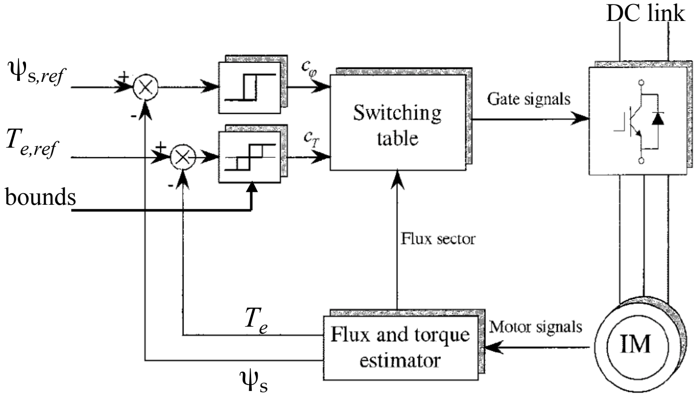
Three-level DC link inverter driving a three-phase induction motor

Control Principle

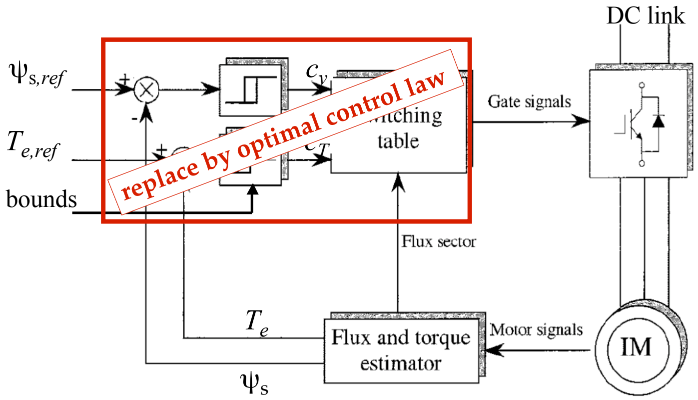


Choose one of 27 voltage vectors (switch combinations) **s.t.:**
- **Torque**, **stator flux** and **NPP** are kept in target window
- Average **switching frequency** is minimized

Classic Direct Torque Control: Control Loop



Optimal Direct Torque Control



Challenge & Solutions

Challenge:

- Combination of very **fast** and **slow dynamics**:
  - switching possible every 25 µs (40 kHz)
  - switching (per stack) done every 2 ms (500 Hz)
  - rotation of field ≈ 20 ms (50 Hz)

===> Very **long prediction horizon** required to capture average switching frequency (several 100 steps)

Solutions:

- **Approximate** average switching frequency by **number of switch transitions** (over horizon)
- **Limit** degrees of freedom
- Low complexity **modeling**

Three MPC Schemes

MPC based on Priority Levels:

- Concept: Three penalty levels, time-varying penalties on switching
- Limit degrees of freedom: Multiple rate model, short horizon
- On-line computation time: > 100 ms

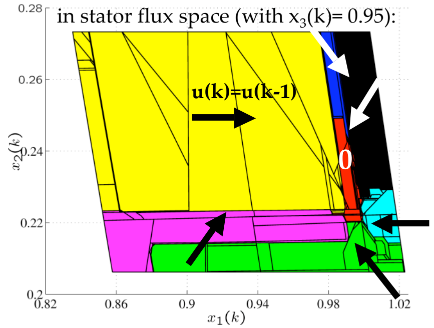
MPC based on Feasibility & Move Blocking:

- Concept: Prioritize feasibility, forward evaluation (in time)
- Limit degrees of freedom: Switching only at k=0 (move blocking)
- On-line computation time: ≈ 1 ms

MPC based on Extrapolation:

- Concept: Extrapolate output trajectories
- Limit degrees of freedom: Switching only at k=0 and k=1
- On-line computation time: ≈ 10 µs

MPC based on Priority Levels: State-feedback Control Law



Control law very complex (47’851 polyhedra)
Performance improvement (reduction of switching freq.) by 25%

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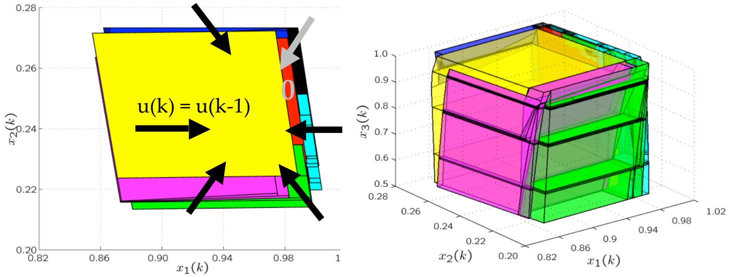
MPC based on Feasibility & Move Block.: State-feedback Control Law

State-feedback control law:

(for $u(k-1) = [1\ -1\ -1]^T$)

in x_1x_2 plane for $x_3 = 0.95$: in $x_1x_2x_3$ space:



colors correspond to control input u(k)

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Complexity and Performance Comparison

	N	Frequency	# Polyhedra
ABB (industrial state of the art)		700-800 Hz	
MPC based on **priority levels**	2	525 Hz	47'851
MPC based on **feasibility** and **move blocking**	2	632 Hz	1192
	3	606 Hz	1891
	5	540 Hz	2907
	7	495 Hz	3737

Simpler control law and improved performance

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Three MPC Schemes

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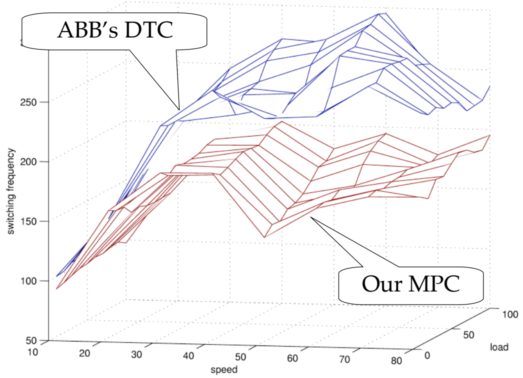
Performance Improvement for all Operating Points



Reduction of switching frequency by up to 45 % (in average 25 %)

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Three MPC Schemes

MPC based on Priority Levels:

- Concept: Three penalt… …ing
- Limit degrees of free…
- On-line computation…

MPC based on Feasib…

- Concept: Prioritize fe…
- Limit degrees of freedom… …g)
- On-line computation tim… …ms

- Control scheme simple, flexible and computationally feasible
- European patent pending
- Implementation by ABB soon

MPC based on Extrapolation:

- Concept: Extrapolate output trajectories
- Limit degrees of freedom: Switching only at k=0 and k=1
- On-line computation time: ≈ 10 μs

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Three MPC Schemes: Summary

MPC based on Priority Levels (… …ion problem in closed form)

- Concept: Three penalty le… …g penalties on switching
- Limit degrees of freed… …e model, horizon short
- On-line computation tin… …0 ms

Very complex, not flexible

MPC based on Feasibility & Mov… …ocking:

- Concept: Prioritize feasibilit… …aluation (in time)
- Limit degrees of freedom… …g (switching only at k=0)
- On-line computation time: …s

Simple, not flexible

MPC based on Extrapolation:

- Concept: Extrapolate output t…
- Limit degrees of freed… …y at k=0 and k=1
- On-line computation tin… …s

Very simple, highly flexible

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Conclusions

- Foundations of a theoretical framework for practical controller design for PWA system have been established
- Complexity reduction and robustness are main research issues
- Applications in industry are beginning
- Software tools are being established

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