Stabilization of Discrete-Time Hybrid Automata

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Outline

- Motivations
- Previous Results
- Stable Convex Combinations in Discrete Time
  - Negative Definite Multiple Stable Convex Combinations
  - Non-Definite Multiple Stable Convex Combinations
  - Common Lyapunov Function
- Numerical Examples
- Conclusions

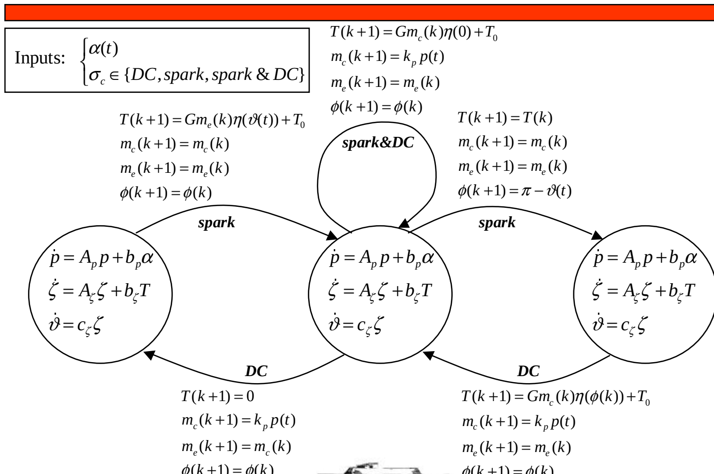
Motivations: Engine's Stabilization

Driving a 4-cylinder-in-line automotive engine to a specified set-point

Hybrid Stabilization Problem

Discrete-Time Formulation

Hybrid Model of the Engine



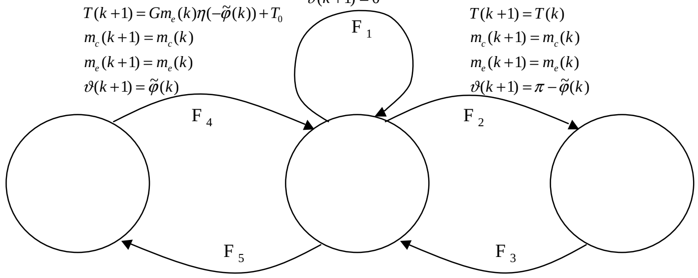
DT-Hybrid Model of the Engine

Inputs: $\begin{cases} m_a(k) \\ \tilde{\varphi}(k) \geq 0 \\ i(k) \in \{1,2,3,4,5\} \end{cases}$

$$\begin{aligned} \zeta(k+1) &= \tilde{A}_\zeta \zeta(k) + \tilde{b}_\zeta T(k) \\ T(k+1) &= G m_c(k) \eta(0) + T_0 \\ m_c(k+1) &= m_a(k) \\ m_e(k+1) &= m_e(k) \\ \vartheta(k+1) &= 0 \end{aligned}$$

$$\begin{aligned} \zeta(k+1) &= \tilde{A}_\zeta \zeta(k) + \tilde{b}_\zeta T(k) \\ T(k+1) &= G m_e(k) \eta(-\tilde{\varphi}(k)) + T_0 \\ m_c(k+1) &= m_c(k) \\ m_e(k+1) &= m_e(k) \\ \vartheta(k+1) &= \tilde{\varphi}(k) \end{aligned}$$

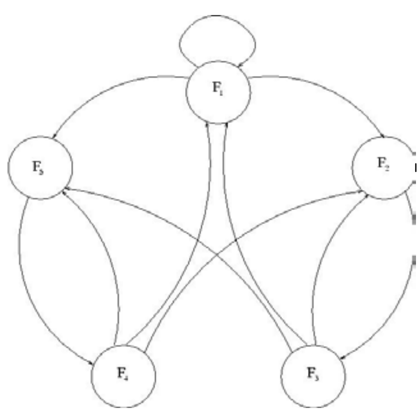
$$\begin{aligned} \zeta(k+1) &= \tilde{A}_\zeta \zeta(k) + \tilde{b}_\zeta T(k) \\ T(k+1) &= T(k) \\ m_c(k+1) &= m_c(k) \\ m_e(k+1) &= m_e(k) \\ \vartheta(k+1) &= \pi - \tilde{\varphi}(k) \end{aligned}$$



$$\begin{aligned} \zeta(k+1) &= \tilde{A}_\zeta \zeta(k) + \tilde{b}_\zeta T(k) \\ T(k+1) &= 0 \\ m_c(k+1) &= m_a(k) \\ m_e(k+1) &= m_c(k) \\ \vartheta(k+1) &= 0 \end{aligned}$$

$$\begin{aligned} \zeta(k+1) &= \tilde{A}_\zeta \zeta(k) + \tilde{b}_\zeta T(k) \\ T(k+1) &= G m_c(k) \eta(\tilde{\varphi}(k)) + T_0 \\ m_c(k+1) &= m_a(k) \\ m_e(k+1) &= m_e(k) \\ \vartheta(k+1) &= 0 \end{aligned}$$

The Hybrid Stabilization Problem



$$\begin{aligned} z(k+1) &= F_{i(k)}(z(k), m_a(k), \tilde{\varphi}(k)) \\ y(k) &= f_{i(k)}(z(k)) \end{aligned}$$

Find the feedback control law:

$$\begin{cases} m_a(z(k), i(k-1)) \\ \tilde{\varphi}(z(k), i(k-1)) \\ i(z(k), i(k-1)) \end{cases}$$

that gives:

$$\lim_{k \to \infty} y(k) = y_0$$

Approaching the Problem: State Stabilization

We start considering the problem of state stabilization for the discrete-time hybrid linear automaton:

$$z(k+1) = A_{i(k)} z(k) + B_{i(k)} u(k)$$

with the same 5-locations FSM.

Find the feedback control law:

$$\begin{cases} u(z(k), i(k-1)) \\ i(z(k), i(k-1)) \end{cases}$$

that gives:

$$\lim_{k \to \infty} z(k) = 0$$

Explicit Design of Stabilizing Switching Strategies

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Previous Results:

- Engine Control:
  - Balluchi, Benvenuti et al. [ACC 2000]: Idle speed control
- Continuous-time Switched Systems:
  - Wicks, Peleties and DeCarlo [CDC 1994]: Stable convex combinations on continuous-time switched systems;
  - Pettersson and Lennartson [ACC 2001]: Stable convex combinations in the context of min-projection strategy;
  - Johansson and Rantzer [IEEE 1998]: Computation of piecewise quadratic Lyapunov functions.
- Discrete-time Switched and Hybrid Systems:
  - Bemporad, Borrelli and Morari [HSCC 2002]: Optimal control of piecewise affine systems;
  - Cuzzola and Morari [HSCC 2001]: Stabilization of piecewise affine systems.

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CT-Stable Convex Combinations

In the continuous-time case, the switched system

$\dot{x} = A_i x$

under the switching law

$i = \operatorname{argmin}_j \left\{ x(t)^T (A_j^T P + P A_j) x(t) \right\}$

is asymptotically stable, with rate of convergence greater or equal to Q, if the following problem has a solution:

$\sum_i \delta_i (A_i^T P + P A_i) \leq -Q$

$\delta_i \geq 0, \quad P, Q > 0$

$\sum_i \delta_i = 1$

DT-Stable Convex Combinations

We propose an extension to the discrete time domain, based on the solution of the system:

$\sum_i \delta_i (A_i^T P A_i - P) \leq -Q$

$\delta_i \geq 0, \quad P, Q > 0$

$\sum_i \delta_i = 1$

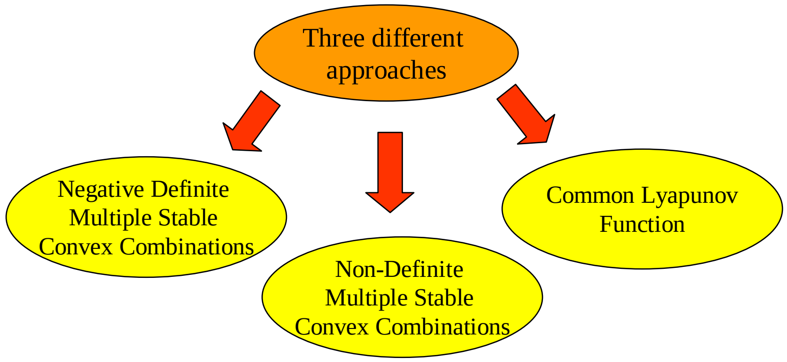
that implies stability for the switched system

$x(k+1) = A_{i(k)} x(k)$

under the switching law

$i(k) = \operatorname{argmin}_j \left\{ x(k)^T (A_j^T P A_j - P) x(k) \right\}$

DT-Stable Convex Combinations and FSMs



Negative Definite Multiple Stable Convex Combinations

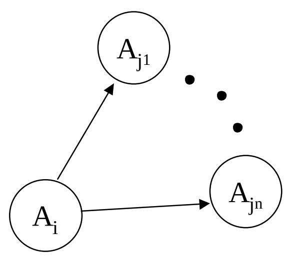
$$\forall i : 1..|L|$$

$$\sum_{j\in\sigma(i)} \delta_{ij}(\tilde{A}_j^T P_j \tilde{A}_j - P_i) \le -Q_i$$

$$\delta_j \ge 0, \quad P_i, Q_j > 0$$

$$\sum_j \delta_{ij} = 1$$

$$\tilde{A}_i = A_i + B_i K_i$$



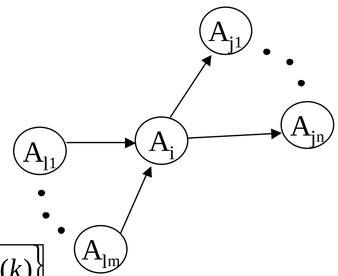
$$i(k) = \arg\min_{j\in\sigma(i)} \left\{ x(k)^T (\tilde{A}_j^T P_j \tilde{A}_j - P_{i(k-1)}) x(k) \right\}$$

$$V(x,k) = x(k)^T P_{i(k-1)} x(k)$$

Non-Definite Multiple Stable Convex Combinations

$$\forall i:1..|L|, \forall l \in \pi(i) \text{ s.t.}:$$
$$x(k-1)^T (\tilde{A}_i^T P_i \tilde{A}_i - P_l) x(k-1) < 0,$$
$$x(k)^T \sum_{j \in \sigma(i)} \delta_{ij} (\tilde{A}_j^T P_j \tilde{A}_j - P_i) x(k) < 0$$
$$\delta_j \geq 0, \quad P_i > 0$$
$$\sum_j \delta_{ij} = 1$$

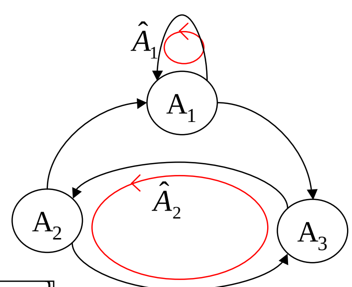
$$i(k) = \arg \min_{j \in Suc(i)} \left\{ x(k)^T (\tilde{A}_j^T P_j \tilde{A}_j - P_{i(k-1)}) x(k) \right\}$$
$$V(x,k) = x(k)^T P_{i(k-1)} x(k)$$



Common Lyapunov Function

$$\sum_i \delta_i (\hat{A}_i^T P \hat{A}_i - P) \leq -Q$$
$$\delta_i \geq 0, \quad P, Q > 0$$
$$\sum_i \delta_i = 1$$
$$\hat{A}_i = \tilde{A}_{i_{(n-1)}} \cdot \tilde{A}_{i_{(n-2)}} \cdot \ldots \cdot \tilde{A}_{i_1}$$
$$\tilde{A}_i = A_i + B_i K_i$$

$$i(k) = \arg\min_j \left\{ x(k)^T (\hat{A}_j^T P \hat{A}_j - P) x(k) \right\}$$
$$V(x) = x^T P x$$



Numerical Examples

Common Lyapunov Function approach on the previous FSM, with the following open-loop dynamic matrices (obtained by a simplified model of the engine):

$$A_1 = \begin{bmatrix} 0.9453 & 3.4842 & 0 \\ 0 & 0 & 3.12 \\ 0 & 0 & 0 \end{bmatrix} \quad b_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 0.9542 & 2.9171 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad b_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0.9907 & 0.5944 & 0 \\ 0 & 0 & 3.9 \\ 0 & 0 & 0 \end{bmatrix} \quad b_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Numerical Examples

with the ‘macro-transformations’ defined as:

$$\hat{A}_1 = \tilde{A}_1; \quad \hat{A}_2 = \tilde{A}_3 \tilde{A}_2$$

The solution obtained is:

$$K_1 = \begin{bmatrix} 1.179 & 5.917 & -0.604 \\ 0.000 & 0.000 & 0.000 \end{bmatrix} \cdot 10^{-6}$$

$$K_2 = \begin{bmatrix} 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} 0.226 & -0.042 & -0.068 \\ -0.229 & -0.018 & -0.619 \end{bmatrix} \cdot 10^{-6}$$